Effects of exit back-up on delamination in drilling composite materials using a saw drill and a core drill

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Abstract

Machining of composites has caught greater attention in manufacturing of structural parts in aerospace, automobile and sporting goods. Composite materials have advantageous features in strength and stiffness coupled with lightweight compared to the conventional metallic materials. Amongst all machining operations, drilling is the most commonly applied method for generating holes for riveting and fastening the structural assembly. Delamination is one of the serious concerns in drilling holes in composite materials at the bottom surface of the workpiece (drill exit). Quite a few references of the drilling of fiber-reinforced plastics report that the quality of cut is strongly dependent on drilling parameter as well as the drill geometry. Saw drills and core drills produce less delamination than twist drills by distributing the drilling thrust toward the hole periphery. Delamination can be effectively reduced or eliminated by slowing down the feed rate when approaching the exit and by using back-up plates to support and counteract the deflection of the composite laminate leading to exit side delaminations. The use of the back-up does reduce the delamination in practice, which its effects have not been well explained in analytical fashion. This paper predicts the effects of backup plate on delamination in drilling composite materials using saw drill and core drill. The critical drilling thrust force at the onset of delamination is calculated and compared with that without backup. The well known advantage of industrial use of backup can be understood fundamentally by the fact that the threshold thrust force at the onset of delamination is increased making the delamination less induced.

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1. Introduction

Amongst all machining operations, drilling using twist drill is the most commonly applied method for generating holes for riveting and fastening structural assemblies [1]. The importance of the drilling process is evident from the many developments in drill design and manufacture in the lasting search for improved drill performance and more economic drill and hole production. For graphite–epoxy or glass–epoxy, a small chisel edge is needed to reduce the thrust force of a twist drill [2,3]. Wu used multifaceted drills to reduce the thrust force up to 70\% as compared to a conventional twist drill has achieved [4]. Doerr et al. designed the drill to cut materials toward the hole center and to shear at the hole edge [5]. Friederich et al. cite the ‘split’ or ‘crankshaft’ point as being very popular in the aircraft and automotive industries [6]. Haggerty and Ernst found that ‘spiral’ point drills performed much better than the conventional ones [7]. The influence of tool wear and the resulted increase of thrust were discussed [8–10]. Okafor and Birdsong found that drill material has the most significant effect on exit hole delamination [11]. Koenig et al. investigated the effect of processing variables on drilling damage [12,13]. A general overview of the various possibilities for composites machining can be found [14].

Although composites are generally fabricated to near-set-shape, the secondary machining operation such as drilling holes is needed. Delamination is one of the major concerns in drilling hole in composite materials at the bottom surface of the workpiece (drill exit). Boeing Aircraft
Co. (Seattle) developed a special milling cutter to address this problem. The size of the delamination zone has been shown to be related to the thrust force developed during the drill process and it is believed that there is a ‘critical thrust force’ below which no damage occurs [12]. Delamination can be greatly reduced or eliminated by reducing feed rates near the exit and using backup plates to support and prevent deformations leading to exit side delaminations. Hocheng and Dharan proposed an analytical model to determine the critical thrust force of the twist drill [15]. Jain and Yang extended this theory by taking into consideration the anisotropy of the composite material. They developed a method for correlating the feed rate with the onset of delamination [2,16]. Hocheng and Tsao [17–19] developed a series of analytical models for various drills (candle stick drill, saw drill, core drill and step drill) for correlating the thrust force at the onset of delamination.

The presence of the backup plate does influence the delamination in practice, but exactly how that works is not analytically understood. This paper presents analysis for the effect of backup plate on delamination in drilling composite materials in the case of using a saw drill and a core drill.

2. Delamination analysis

In drilling of composite laminates, the uncut thickness to withstand the drilling thrust force decreases as the drill approaches to the exit plane. The laminate at the bottom can get separated from its interlaminar bond around the hole. At some point the drilling load exceeds the interlaminar bond strength and delamination occurs. One approach was to perform the experimental work and hold the machinability data to account for delamination of different materials for various tools and machining parameters [12]. In consideration of the expense of time and cost, an analytical approach is endeavored to embody the aspects of materials and process parameters in a model to predict the onset of delamination [20].

2.1. Physical model

At the propagation of delamination, the drill movement of distance dX is associated with the work done by the thrust force FA, which is used to deflect the plate as well as to propagate the interlaminar crack. The energy balance equation gives

\[ G_{IC} \, dA = F_A \, dX - dU \]  

where dU is the infinitesimal strain energy, dA is the increase in the area of the delamination crack, and \( G_{IC} \) is the critical crack propagation energy per unit area in mode I. The value of \( G_{IC} \) is assumed a constant to be a mild function of strain rate by Saghizadeh and Dharan [21].

A back-up support underneath the workpiece can counteract the downward bending deflection of the laminae caused by the drilling thrust force. The support material is in full contact with the bottom of workpiece initially, namely the reactions are a uniform upward load is applied to the back side of workpiece. However, when the drilling proceeds, the laminate will be inevitably slightly deflected. The support body has much higher stiffness than the thin laminae being bent, thus will not fully conform to the laminate deflection. Meanwhile, the internal reaction force associated with the downward bending of the uncut laminate in the laminate will lift up the laminate from the outskirt of the circular crack, leaving the bottom side of workpiece in approximately point contact with the back-up material. Namely the uniform load changes to a concentrated load applied on the bottom center of the workpiece. The deflected laminae are subject to both drilling thrust on entry side and back-up force from exit side.

2.2. Mathematical analysis

2.2.1. Drilling with saw drill without backup [17]

Fig. 1 depicts the schematics of delamination without backup plate for saw drill of diameter c. Saw drill can acquire better machining quality in drilling composite laminates. One reason is that the saw drill utilizes the peripheral distribution of thrust for drilling the composite laminates. In Fig. 1, \( F_S \) is the thrust force, \( X \) is the displacement, \( H \) is the workpiece thickness, \( h \) is the uncut depth under tool, and \( a \) is the radius of delamination. The isotropic behavior and pure bending of the laminate are assumed in the model.

In Fig. 1, the deflection of the circular plate is given [22]

\[ X = \frac{F_S}{8\pi M} \left[ \left( r^2 + c^2 \right) \ln \frac{c}{a} + \frac{1}{2} \left( 1 - \frac{c^2}{a^2} \right) (a^2 + r^2) \right] \]  

Fig. 1. Circular plate model of delamination for saw drill without backup.
(ii) \( c \leq r \leq a \) (outer portion)

\[
X = \frac{F_s}{8\pi M} \left[ (r^2 + c^2) \ln \frac{r}{a} + \frac{1}{2} \left( 1 + \frac{c^2}{a^2} \right) (a^2 - r^2) \right]
\]  

(3)

where \( r \) is radial coordinate in polar coordinates when isotropic circular plate (solid or annular) is subjected to rotationally symmetric loads, \( M = Eh^3/12(1 - \nu^2) \) = flexural rigidity of the plate, \( E \) is Young’s modulus, and \( \nu \) is Poisson’s ratio.

The stored strain energy is as following

(i) \( 0 \leq r \leq c \) (inner portion)

\[
U_1 = \pi \int_0^c M \left( \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right)^2 r \, dr
\]

\[
= \frac{F_s}{16\pi M} \left[ 2c^2 \ln^2 \frac{c}{a} + 2c^2 \ln \frac{c}{a} - 4c^4 \ln \frac{c}{a}
+ \frac{c^2}{2} - \frac{c^4}{2a^2} + \frac{c^6}{2a^4} \right]
\]

(4)

(ii) \( c \leq r \leq a \) (outer portion)

\[
U_2 = \pi \int_c^a M \left( \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right)^2 r \, dr
\]

\[
= \frac{F_s}{16\pi M} \left[ \frac{a^2}{2} - \frac{c^2}{2} + \frac{c^4}{2a^2} - \frac{c^6}{2a^4}
- 2c^2 \ln^2 \frac{c}{a} + \frac{2c^4}{a^2} \ln \frac{c}{a} \right]
\]

(5)

The total strain energy is

\[
U = U_1 + U_2 = \frac{F_s}{32\pi M} \left( 4c^2 \ln \frac{c}{a} + a^2 - \frac{c^4}{a^2} \right)
\]

(6)

Differentiation of Eq. (6) with respect to \( a \) yields

\[
\frac{dU}{da} = \frac{F_s}{16\pi M} \left( a - \frac{2c^2}{a} + \frac{c^4}{a^3} \right)
\]

(7)

The critical thrust force \( (F_s) \) at the onset of crack propagation can be calculated

\[
F_s = \pi \sqrt{\frac{32G_{IC}M}{(1 - 2\nu^2 + \nu^4)}}
\]

(8)

where \( s = \text{cr}a \).

### 2.2.2. Drilling with saw drill with backup

Fig. 2 depicts the center of the circular plate is supported under the workpiece with a saw drill-induced delamination. In Fig. 2, \( F_{SF} \) is the upward reaction force from the backup plate equal to the downward thrust force of the drill. The isotropic infinite plate behavior and pure bending of the laminate are assumed in the model.

The deflection of the circular plate is given [22]

(i) \( 0 \leq r \leq c \) (inner portion)

\[
X = \frac{F_{SF}c^2}{8\pi M} \left[ 1 - \frac{c^2}{a^2} - \ln \frac{c}{a} + \frac{c^2}{a^2} \ln \frac{c}{a} - \ln \frac{r}{a} + \frac{2c^4}{a^2} \ln \frac{c}{a} \right]
\]

(9)

(ii) \( c \leq r \leq a \) (outer portion)

\[
X = \frac{F_{SF}c^2}{8\pi M} \left[ 1 - \frac{r^2}{a^2} - \ln \frac{c}{a} + \frac{r^2}{a^2} \ln \frac{c}{a} + \ln \frac{r}{a} \right]
\]

(10)

Differentiation of Eq. (9) with respect to \( da \) yields

\[
\frac{dX}{da} = \frac{F_{SF}}{2\pi M} \left( \frac{c^2 \ln^2 \frac{c}{a}}{a^3} \right)
\]

(11)

The stored strain energy is as following

(i) \( 0 \leq r \leq c \) (inner portion)

\[
U_1 = \pi \int_0^c M \left( \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right)^2 r \, dr
\]

\[
= \frac{F_{SF}c^2}{4\pi M} \left[ \frac{c^2}{4} - 2\frac{c^4}{2a^2} + \frac{6c^6}{4a^4}
+ \left( \frac{2c^6}{a^4} - \frac{c^4}{a^2} \right) \ln \frac{c}{a} - \frac{2c^6}{a^4} \ln^3 \frac{c}{a} + \frac{2c^6}{a^2} \ln^2 \frac{c}{a} \right]
\]

(12)

(ii) \( c \leq r \leq a \) (outer portion)

\[
U_2 = \pi \int_c^a M \left( \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right)^2 r \, dr
\]

\[
= \frac{F_{SF}c^4}{4\pi M} \left[ \frac{1}{4a^2} - \frac{c^2}{4a^2} + \frac{c^4}{4a^4} - \frac{1}{a^2} \ln \frac{c}{a}
+ \left( \frac{1}{2a^2} - \frac{2c^2}{a^4} \right) \ln^2 \frac{c}{a} + \frac{2c^2}{a^4} \ln^3 \frac{c}{a} - \frac{2c^2}{a^2} \ln^4 \frac{c}{a} \right]
\]

(13)
The total strain energy is
\[
U = U_1 + U_2 = \frac{F_{SF}^2}{4\pi M} \left[ \frac{c^2}{4} - \frac{c^4}{4a^2} + \frac{c^4}{2a^2} \ln \frac{c}{a} - \frac{c^4}{2a^2} \ln^2 \frac{c}{a} \right]
\] (14)

Differentiation of Eq. (14) with respect to \(a\) yields
\[
dU \bigg|_{a} = \frac{F_{SF}^2}{4\pi M} \left( \frac{c^4}{a^3} \ln \frac{c}{a} \right)
\] (15)

The critical thrust force \(F_{SF}\) at the onset of crack propagation can be calculated
\[
F_{SF} = \frac{\pi}{2s^2|\ln s|} \sqrt{32G_IcM}
\] (16)
where \(s = c/a\). The comparison of \(F_{SF}\) and \(F_S\) in Eqs. (16) and (8) gives
\[
\frac{F_{SF}}{F_S} = \frac{\sqrt{1 - 2s^2 + s^4}}{2s^2|\ln s|}
\] (17)

### 2.2.3. Drilling with core drill without backup [17]

Fig. 3 depicts the schematics of a core drill and the induced delamination. The outer and inner deflection of a given by Rudolph [22]. It provides more rigorous approach than the previous work [17,19].

(i) \(0 \leq r \leq c\) (inner portion)
\[
X = \frac{F_R}{64\pi M} \left[ 4a^2 - 3c^2(2 - 2\beta + \beta^2) \right]
- \frac{2r^2c^2(2 - 2\beta + \beta^2)}{a^2} + 4r^2c^2(2 - 2\beta + \beta^2) \frac{\ln c}{a}
- \frac{4c^2(1 - \beta)^2}{\beta(2 - \beta)} \ln (1 - \beta) + 8r^2 \frac{c}{a}
- \frac{8(1 - \beta)^2}{\beta(2 - \beta)} \ln (1 - \beta)
\] (18)

(ii) \(c \leq r \leq a\) (outer portion)
\[
X = \frac{F_R}{32\pi M} \left[ 2a^2 - 2r^2 + 2c^2(2 - 2\beta + \beta^2) \frac{r}{a} \right]
+ 4r^2 \ln \frac{r}{a} + c^2(2 - 2\beta + \beta^2) - \frac{r^2c^2(2 - 2\beta + \beta^2)}{a^2}
\] (19)

where \(F_R\) is the thrust force of the core drill. \(c\) is the outer radius of core drill. \(r\) is the thickness of core drill, and \(\beta\) is the ratio between thickness and radius of core drill (namely, \(\beta = t/c\)).

Differentiation of Eq. (18) with respect to \(da\) yields
\[
dX \bigg|_{a} = \frac{F_R}{64\pi M} \left[ 8a - 8\frac{r^2}{a} - 4c^2(2 - 2\beta + \beta^2) \right.
+ \left. 4r^2c^2(2 - 2\beta + \beta^2) \frac{1}{a^3} \right]
\] (20)

The stored strain energy is

(i) \(0 \leq r \leq c\) (inner portion)
\[
U_1 = \pi \int_0^c M \left[ \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right]^2 r \, dr
= \frac{F_R^2}{64\pi M} \left[ \frac{c^6(2 - 2\beta + \beta^2)^2}{2a^4} + 8c^2 \frac{\ln^2 c}{a}
+ 8c^2 \frac{(1 - \beta)^4}{\beta^2(2 - \beta)^2} \ln^3 (1 - \beta) - 4c^2 \frac{(2 - 2\beta + \beta^2)}{a^2} \frac{\ln c}{a}
- 4c^2 \frac{(1 - \beta)^2(2 - 2\beta + \beta^2)}{a^2 \beta(2 - \beta)} \ln (1 - \beta)
+ \frac{16c^2(1 - \beta)^2}{\beta(2 - \beta)} \ln (1 - \beta) \ln c \frac{1}{a} \right]
\] (21)

(ii) \(c \leq r \leq a\) (outer portion)
\[
U_2 = \pi \int_c^a M \left[ \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right]^2 r \, dr
= \frac{F_R^2}{64\pi M} \left[ 2a^2 - 2c^2 + \frac{c^4(2 - 2\beta + \beta^2)^2}{2a^4}
- 8c^2 \frac{\ln^2 c}{a} + 4c^2 \frac{(2 - 2\beta + \beta^2)}{a^2} \frac{\ln c}{a}
- \frac{c^6(2 - 2\beta + \beta^2)^2}{2a^4} \right]
\] (22)
The total strain energy is
\[ U = U_1 + U_2 \]
\[ = \frac{F_R^2}{64\pi M} \left[ 2a^2 - 2c^2 + \frac{c^2(2 - 2\beta + \beta^2)^2}{2a^2} \right. \]
\[ + \frac{8c^2(1 - \beta)^2}{\beta^2(2 - \beta)^3}\ln(1 - \beta) \]
\[ - \frac{4c^4(1 - \beta)^2(2 - 2\beta + \beta^2)}{a^2\beta(2 - \beta)}\ln(1 - \beta) \]
\[ + \frac{16c^2(1 - \beta)^2}{\beta(2 - \beta)}\ln(1 - \beta)\ln\left(\frac{c}{a}\right) \]  
(23)

and
\[ \frac{dU}{da} = \frac{F_R^2}{16\pi M} \left[ a - \frac{c^2(2 - 2\beta + \beta^2)^2}{4a^3} \right. \]
\[ + \frac{2c^4(1 - \beta)^2(2 - 2\beta + \beta^2)}{a^2\beta(2 - \beta)}\ln(1 - \beta) \]
\[ + \frac{4c^2(1 - \beta)^2}{a\beta(2 - \beta)}\ln(1 - \beta)\]  
(24)

One obtains the thrust force of the core drill at the onset of crack propagation as
\[ F_R = \pi \left\{ \alpha \left[ 1 - \left( 4 - 4\beta + \frac{\beta^2}{2} \right) + \frac{4(1 - \beta)^2}{a^2\beta(2 - \beta)}\ln(1 - \beta) \right] \right\}^{32GicM} \]
\[ \frac{1}{2} \left[ \frac{2(1 - \beta^2)(2 - 2\beta + \beta^3)}{a^2(2 - \beta)}\ln(1 - \beta) \right]^{1/2} \]  
(25)

2.2.4. Drilling with core drill with backup

Fig. 4 depicts the center of the circular plate is supported under the workpiece with a core drill-induced delamination. In Fig. 4, \( F_{RF} \) is the upward reaction force from the backup plate equal to the downward thrust force of the drill. The isotropic infinite plate behavior and pure bending of the laminate are assumed in the model.

\[ X = \frac{F_{RF}^2}{64\pi M} \left( k_0 + k_1 \ln\left(\frac{c}{a}\right) + k_2 \ln\left(\frac{c}{a}\right)\ln\left(\frac{r}{a}\right) + k_3 \ln\left(\frac{r}{a}\right) \right) \]  
(26)

where
\[ k_0 = 4 - \frac{5c^2(2 - 2\beta + \beta^2)}{a^2} - \frac{8(1 - \beta)^2}{\beta(2 - \beta)}\ln(1 - \beta) \]
\[ k_1 = 8 + \frac{4c^4(1 - \beta)^4}{a^2\beta(2 - \beta)}\ln(1 - \beta) \]
\[ k_2 = -\frac{8c^2(2 - 2\beta + \beta^2)}{a^2} \]
\[ k_3 = -8 + \frac{6c^2(2 - 2\beta + \beta^2)}{a^2} + \frac{8c^2(1 - \beta)^4}{a^2\beta(2 - \beta)}\ln(1 - \beta) \]

(ii) \( c \leq r \leq a \) (outer portion)

\[ X = \frac{F_{RF}^2}{64\pi M} \left( k_4 + k_5 \ln\left(\frac{c}{a}\right) + k_6 \ln\left(\frac{c}{a}\right)\ln\left(\frac{r}{a}\right) + k_7 \ln\left(\frac{r}{a}\right) \right) \]  
(27)

where
\[ k_4 = 5(2 - 2\beta + \beta^2)\left(1 - \frac{r^2}{a^2}\right) \]
\[ + \frac{4(1 - \beta)^4}{\beta(2 - \beta)}\ln(1 - \beta)\left(1 - \frac{r^2}{a^2}\right) \]
\[ k_5 = -4(2 - 2\beta + \beta^2)\left(1 - \frac{r^2}{a^2}\right) \]
\[ k_6 = -\frac{8r^2(2 - 2\beta + \beta^2)}{a^2} \]
\[ k_7 = 2(2 - 2\beta + \beta^2)\left(2 + \frac{3r^2}{a^2}\right) \]
\[ + \frac{8r^2(1 - \beta)^4}{a^2\beta(2 - \beta)}\ln(1 - \beta) \]

Differentiation of Eq. (26) with respect to \( da \) yields
\[ \frac{dX}{da} = \frac{F_{RF}}{16\pi M} \left( \frac{c^4}{a^3} k_4 \right) \]  
(28)
\( k_8 = (2 - \beta)^2 \left[ -\frac{1}{4} (2 - 2\beta + \beta^2) \ln \frac{c(1 - \beta/2)}{a} \right. \)
\[ \left. - \frac{(1 - \beta)^4}{\beta(2 - \beta)} \ln(1 - \beta) \ln \frac{c(1 - \beta/2)}{a} \right. \]
\[ \left. + (2 - 2\beta + \beta^2) \ln \frac{c}{a} - \frac{c(1 - \beta/2)}{a} \right] \]

The stored strain energy is as following

(i) \( 0 \leq r \leq c \) (inner portion)

\[ U_1 = \pi \int_0^c M \left( \frac{d^2 X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right) \frac{2}{r} dr \]
\[ = \frac{F_{eq}^2 c}{16\pi M} \left\{ k_9 + \frac{c^2}{a^2} \left[ k_{10} + k_{11} \ln \frac{c}{a} + k_{12} \ln^2 \frac{c}{a} \right. \right. \]
\[ \left. \left. + k_{13} \ln^3 \frac{c}{a} + k_{14} \ln^4 \frac{c}{a} \right] \right\} \]
\[ \tag{29} \]

\[ k_9 = \frac{1}{2} + \frac{2(1 - \beta)^4}{\beta(2 - \beta)} \]

\[ k_{10} = (2 - 2\beta + \beta^2) \left[ -\frac{3}{4} + \frac{(1 - \beta)^2}{\beta(2 - \beta)} \ln(1 - \beta) \right. \]
\[ \left. - \frac{(1 - \beta)^4}{\beta(2 - \beta)} \ln(1 - \beta) + \frac{c^2}{a^2} \left[ \frac{13}{32} (2 - 2\beta + \beta^2)^2 \right. \] \[ \left. \left. + \frac{3}{8} (2 - 2\beta + \beta^2) \ln(1 - \beta) \right. \right. \]
\[ \left. \left. + \frac{1}{2\beta(2 - \beta)^2} \ln^2(1 - \beta) \right] \right. \]

\[ k_{11} = (2 - 2\beta + \beta^2) \left[ 1 - \frac{3(1 - \beta)^2}{\beta(2 - \beta)} \ln(1 - \beta) \right. \]
\[ \left. - \frac{4(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) - \frac{c^2}{a^2} \left[ \frac{3}{2} (2 - 2\beta + \beta^2)^2 \right. \] \[ \left. \left. + \frac{1}{2\beta(2 - \beta)^2} (1 + (1 - \beta)^2) \ln(1 - \beta) \right] \right. \]

\[ k_{12} = \frac{4(1 - \beta)^6}{\beta(2 - \beta)} \ln(1 - \beta) \]
\[ + \frac{c^2}{a^2} \left[ \frac{21}{8} (2 - 2\beta + \beta^2)^2 \right. \]
\[ + \frac{6(1 - \beta)^6}{\beta(2 - \beta)} \ln(1 - \beta) \]
\[ + \frac{2(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \] \]

\[ k_{13} = -\frac{c^2}{a^2} \left[ (2 - 2\beta + \beta^2)^3 \right. \]
\[ \left. - \frac{4(1 - \beta)^6}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \right] \]

\[ k_{14} = \frac{2c^2}{a^2} (2 - 2\beta + \beta^2)^2 \]

(ii) \( c \leq r \leq a \) (outer portion)

\[ U_2 = \frac{1}{2} \int_c^a \left( M \frac{d^2 X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right) \frac{2}{r} dr \]
\[ = \frac{F_{eq}^2 c}{16\pi M} \left\{ k_{15} + k_{16} \ln \frac{c}{a} + k_{17} \ln^2 \frac{c}{a} \right. \]
\[ \left. + k_{18} \ln^3 \frac{c}{a} + k_{19} \ln^4 \frac{c}{a} \right\} \]
\[ \tag{30} \]

\[ k_{15} = \frac{13}{32} (2 - 2\beta + \beta^2)^2 \]
\[ + \frac{3(1 - \beta)^4}{4\beta(2 - \beta)} (2 - 2\beta + \beta^3) \ln(1 - \beta) \]
\[ + \frac{(1 - \beta)^8}{2\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \left( 1 - \frac{c^2}{a^2} \right) \]

\[ k_{16} = -\frac{3}{4} (2 - 2\beta + \beta^2)^2 \]
\[ - \frac{(1 - \beta)^4}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \]
\[ + \frac{c^2}{a^2} \left[ \frac{3}{2} (2 - 2\beta + \beta^2)^2 + \frac{2(1 - \beta)^6}{\beta(2 - \beta)} \right. \]
\[ \left. \times (2 - 2\beta + \beta^2) \ln(1 - \beta) \right] \]

\[ k_{17} = \frac{1}{2} (2 - 2\beta + \beta^2)^2 \]
\[ - \frac{c^2}{a^2} \left( \frac{21}{8} (2 - 2\beta + \beta^2)^2 \right. \]
\[ \left. + \frac{6(1 - \beta)^6}{\beta(2 - \beta)} (2 - 2\beta + \beta^3) \ln(1 - \beta) \right. \]
\[ \left. + \frac{2(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \right] \]

\[ k_{18} = \frac{c^2}{a^2} \left[ \frac{3}{2} (2 - 2\beta + \beta^2)^2 \right. \]
\[ \left. + \frac{4(1 - \beta)^6}{\beta(2 - \beta)} (2 - 2\beta + \beta^3) \ln(1 - \beta) \right] \]
\[ k_{19} = -\frac{2c^2}{a^2} (2 - 2\beta + \beta^2)^2 \]

The total strain energy is

\[ U = U_1 + U_2 \]
\[ = \frac{F_{eq}^2 c}{16\pi M} \left\{ k_9 + \frac{c^2}{a^2} \left[ k_{20} + k_{21} \ln \frac{c}{a} + k_{22} \ln^2 \frac{c}{a} \right] \right\} \]
\[ \tag{31} \]
\[ k_{20} = -\frac{3}{4} (2 - 2\beta + \beta^2) + \frac{13}{32} (2 - 2\beta + \beta^2)^2 \\
+ \frac{(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{(1 - \beta)^4}{\beta(2 - \beta)^2} \ln(1 - \beta) \\
+ \frac{3(1 - \beta)^4}{4\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
+ \frac{(1 - \beta)^8}{2\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \]

\[ k_{21} = (2 - 2\beta + \beta^2) - \frac{3}{4} (2 - 2\beta + \beta^2)^2 \\
- \frac{3(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{(1 - \beta)^4}{\beta(2 - \beta)^2} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{4(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \]

\[ k_{22} = \frac{1}{2} (2 - 2\beta + \beta^2)^2 + \frac{4(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \]

Differentiation of Eq. (31) with respect to \( a \) yields

\[ \frac{dU}{da} = -\frac{F_{RF}^2}{16\pi M a} \left[ \frac{c^4}{a^4} \left( k_{23} + k_{24} \ln \frac{c}{a} + k_{25} \ln^2 \frac{c}{a} \right) \right] \]

\[ k_{23} = -\frac{1}{2} (2 - 2\beta + \beta^2) + \frac{1}{16} (2 - 2\beta + \beta^2)^2 \\
- \frac{(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{2(1 - \beta)^4}{\beta(2 - \beta)^2} \ln(1 - \beta) \\
+ \frac{(1 - \beta)^8}{2\beta^2(2 - \beta)^2} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{4(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \]

\[ k_{24} = 2(2 - 2\beta + \beta^2) - \frac{1}{2} (2 - 2\beta + \beta^2)^2 \\
+ \frac{2(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{2(1 - \beta)^4}{\beta(2 - \beta)^2} (2 - 2\beta + \beta^2) \ln(1 - \beta) \\
- \frac{8(1 - \beta)^6}{\beta^2(2 - \beta)^2} \ln^2(1 - \beta) \]

\[ k_{25} = (2 - 2\beta + \beta^2)^2 + \frac{8(1 - \beta)^2}{\beta(2 - \beta)} (2 - 2\beta + \beta^2) \ln(1 - \beta) \]

The critical thrust force \( (F_{RF}) \) at the onset of crack propagation can be calculated

\[ F_{RF} = \frac{\pi}{4} \frac{32G_{IC}}{(k_8 + k_{23} + k_{24} \ln s + k_{25} \ln^2 s)^4} \]

where \( s = c/a \). The comparison of \( F_{RF} \) and \( F_S \) in Eqs. (33) and (25) gives

\[ \frac{F_{RF}}{F_S} = \sqrt{\frac{1 - 2s^2 + s^4}{(k_8 + k_{23} + k_{24} \ln s + k_{25} \ln^2 s)^4}} \]

3. Experimental set-up

3.1. Specimen

Composite laminates are made from the woven WFC200 fabric carbon fiber by autoclave molding. The stacking sequence of the laminates was [0/90]_8S. The plates are cut into coupon specimens of 60 mm by 60 mm. Sixteen laminas make the plate thickness 4 mm. The fiber volume fraction is 0.55, the modulus of elasticity \( (E_s) \) is 18.4 GPa, the energy release rate \( (G_{IC}) \) is 140 J/m² and the Poisson ratio \( (\nu) \) is 0.3.

With backup drilling, an aluminum plate of 2 mm thickness is placed under the specimen; while without backup drilling, there is nothing under the specimen, which is free to bend under the drill load.

3.2. Drilling test

Drilling tests were carried out on a LEADWELL MCV-610AP vertical machining center in which the thrust force was measured with a Kistler 9273 piezoelectric dynamometer. The force signals were transmitted to Kistler 5007 charge amplifier and stored on a TEAC DR-F1 digital recorder subsequently. The drilling set-up used in data collection is shown in Fig. 5. A proper fixture with a center hole of 24 mm diameter was used to support the laminate, which is firmly held on top of the dynamometer. The saw drill of high speed steel 10 mm in diameter was used. The core drills are 10 mm in diameter plated with diamond of a #60 grit size at front end. All drilling tests were conducted coolant free at a spindle speed of 1000 rpm and feed rates of 8, 12 and 16 mm/min.

4. Results and discussion

A comparison of the theoretical critical thrust force of saw drill with and without backup as a function of the drill diameter and the crack size is shown in Fig. 6. The thrust
force of saw drill with and without backup remains little different at \( s \geq 0.9 \) (i.e. the delamination size \( (a) \) is close to the drill \( (c) \)). The thrust force of saw drill with backup increases fast with its maximum at \( s = 0 \). \( s = 0 \) is the case approaching the use of twist drill that exerts a central point load on the laminate. At \( s = 1 \), the drilling thrust is completely distributed to the periphery of the extent of delamination making it quite hard to propagate the crack. Hence the critical thrust force at the onset of delamination growth approaches to theoretical infinite whether the backup is used to support the laminate or not. The critical thrust force in use of the backup has a minimum at a certain \( s \) between 0 and 1.

As pointed out by DiPaolo et al. [23], the delamination of size less than the drill \( (s < 1) \) is not of concern because it is drilled out afterwards anyway. When the delamination grows beyond the drill radius, the saw drill with backup...
plate can sustain much larger thrust force than the saw drill without backup plate. Hence, the backup plate can offer higher feed rate to drill the composite materials without causing delamination. The result is in good agreement with the industrial experience.

Fig. 7 shows the predicted critical thrust force for the core drill with and without backup. Fig. 7 indicates that for a given $\beta$, the thrust force of a core drill with or without backup plate remains little different at large $s$ (i.e. the drill is large compared with the delamination). This fact in the range of large $s$ was discussed in Section 3 of the saw drill case. The physics remains the same. As the $s$ increases, the thrust force increases whether the backup is used or not. Fig. 7 illustrates the thrust force of core with backup is much higher than core drill without backup at small $s$, where the delamination growth can be effectively counteracted by the support from the backup. Fig. 8 shows the correlation between the measured thrust force and feed rate for saw drill and core drill with and without backup plate. The drilling thrust of saw drill and core drill with backup plate in the tested range of feed rate is higher than saw drill and core drill without backup plate, respectively, due to the resistance provided by the backup. It does not imply that saw drill and core drill with backup plate is more susceptible to delamination damage in drilling. Fig. 8 reveals the critical thrust force of saw drill and core drill without backup plate lies at the similar level of the measured drilling thrust, while the critical thrust force of either saw drill or core drill with backup plate is much larger than the measured thrust in operation. The figure illustrates the safety margin of the process window for both saw drill and core drill gains significantly with the backup in practice. Higher drilling efficiency is also achieved by the allowable larger feed rate.

5. Conclusions

The effects of backup on the delamination caused by saw drill and core drill in drilling of composite materials have been well known in industry. Their analytical models are presented in the study. The theoretical results are obtained based on classical elasticity, linear elastic fracture mechanics and energy conservation law. Based on the proposed models, both the saw drill and the core drill with backup offer a critical thrust force than those without backup. The results agree with the industrial experience, that the drill allows for larger critical thrust force can be operated at larger feed rate without delamination damage. This approach of mechanics can be extended to examine the effects of backup in used of various drills and the design of the associated backup.

References


Fig. 9 clearly indicates the effectiveness in use of backup plate for the saw drill and the core drill. The current analysis explains why to use the backup plate to assist quality drilling of composite material.
Delamination in Drilling Advanced Fiber Reinforced Composites,