Evaluation of anisotropic thermal conductivity for unidirectional FRP in laser machining

C.T. Pan, H. Hocheng*

Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu, Taiwan, ROC

Received 20 October 1998; received in revised form 1 May 2000; accepted 17 May 2000

Abstract

Due to the significant thermal anisotropy and the high-ablation temperature of carbon fiber, thermal damage can be produced during the thermal processing of carbon fiber-reinforced plastics by laser. It leads to poor assembly tolerance and long-term performance deterioration. The current study investigates the grooving of unidirectional Carbon/Epoxy in several representative directions relative to the fiber axis. Two methods, the Eigenvalue Method and the Isotherm Method, have been used to evaluate the general anisotropic thermal conductivities from the principal thermal conductivities. A finite-difference analysis, incorporating the Conductivity Ellipsoid Model due to Jaeger and the developed thermal conductivity models, is adopted to determine the extent of heat affected zone (HAZ). Simulations using either thermal conductivity model show a reasonable agreement with experimental results, i.e. the Eigenvalue Method and Isotherm Method for determining the thermal conductivity in generic direction are equally effective in the subsequent prediction of HAZ of an unidirectional FRP.

Copyright © 2001 Elsevier Science Ltd. All rights reserved.

Keyword: Heat affected zone

1. Introduction

Composite materials possess advantages in structural application as a result of their high-specific strength and other directional properties. They have been used widely for important components. Although composites are often cured to final shape, machining can be required still at both the prepreg and product stages. Conventional machining operations are difficult due to the anisotropy, inhomogeneous composition and abrasive reinforcement of composite material. Excessive tool wear significantly increases the machining time and cost. Laser machining using high-power density beam offers several advantages over conventional methods, such as no tool wear and no contact force-induced material disintegration. Besides, the absorbability of laser beam is good for most composites. However, since laser machining is a thermal process, and the mechanical properties strongly depend on temperature, a structural component cannot be designed safely without considering the temperature field during the process. Reliable thermal conductivity data is an essential input for such an analysis.

The related research on laser-induced heat affected zone (HAZ) in composite is, however, limited to experimental work and isotropic analysis. Tagliaferri et al. (1987) [2] reveal that the extent of the HAZ depends strictly on feed rate. The higher the speed of the laser beam, the smaller the volume of damage and the better the cut finish. Graphite-reinforced composites are found to be less suitable for laser cutting due to high-fiber conductivity and vaporization temperature [1,2]. An isotropic analysis of the HAZ was conducted for the composite materials. Comparison between the predictions and experimental results was presented [3].

A few theories are available for predicting the thermal conductivity of unidirectional fiber-reinforced plastics (FRP) in the direction parallel to and perpendicular to fiber axis. These theories assume a knowledge of fiber and matrix properties and volume fractions, interphase properties, and fiber architecture. The current paper recalls some of the well-known theories as follows:

\[ k_1 = \frac{V_{\text{fiber}} k_{\text{fiber}} + V_{\text{matrix}} k_{\text{matrix}}}{V_{\text{fiber}}} \]  

(1a)

for the direction parallel to fiber axis and

\[ \frac{1}{k_2} = \frac{V_{\text{fiber}}}{k_{\text{fiber}}} + \frac{V_{\text{matrix}}}{k_{\text{matrix}}} \]  

(1b)

for the direction perpendicular to fiber axis [10] where \( V_{\text{matrix}} \)
Table 1
Measured thermal properties of carbon/epoxy

<table>
<thead>
<tr>
<th>Material</th>
<th>Density ($\rho$) kg/m$^3$</th>
<th>Specific heat (C) J/kgK</th>
<th>Conductivity ($k$) W/mK</th>
<th>Diffusivity ($\alpha$) m$^2$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parallel to Fiber Axis</td>
<td>45° to Fiber Axis</td>
<td>Perpendicular to Fiber Axis</td>
<td></td>
</tr>
<tr>
<td>Carbon/Epoxy</td>
<td>1530</td>
<td>4.50</td>
<td>2.95</td>
<td>0.67</td>
</tr>
</tbody>
</table>

and $k_{\text{matrix}}$ denote the volume fraction and the thermal conductivity of the matrix, while, $V_{\text{fiber}}$ and $k_{\text{fiber}}$ stand for the corresponding properties of the fibers.

However, the prediction for thermal conductivity in the generic direction (neither parallel nor perpendicular to fibers) is currently unavailable. The present study provides the modeling of the effective anisotropic thermal conductivity. Furthermore, a numerical heat conduction model using the anisotropic heat conductivity instead of a single isotropic conductivity in particular directions for the thermal analysis is presented. The experiment shows that the effects of fiber orientation on HAZ can be predicted by the proposed methods.

2. Thermal conductivity in general directions

2.1. Eigenvalue method

When an anisotropic composite is grooved (by laser machining) along a non-principal direction, the cross-derivative coefficients of thermal conductivity will be involved in the differential equation of heat conduction. The thermal conductivity of an anisotropic solid is composed of nine conductivity coefficients in a second-order symmetric tensor

$$
\mathbf{k} = \begin{bmatrix}
k_{11} & k_{12} & k_{13} \\
k_{21} & k_{22} & k_{23} \\
k_{31} & k_{32} & k_{33}
\end{bmatrix}
$$

(2a)

To solve the cross-derivative terms of thermal conductivity, this research presents the Eigenvalue method

$$
\begin{bmatrix}
k_{11} - \lambda_1 & k_{12} & k_{13} \\
k_{21} & k_{22} - \lambda_2 & k_{23} \\
k_{31} & k_{32} & k_{33} - \lambda_3
\end{bmatrix} = 0
$$

(2b)

where $\lambda_1$, $\lambda_2$, and $\lambda_3$ are the principal-axis thermal conductivity, i.e. $k_1$, $k_2$, and $k_3$.

The coordinate axes ($x,y,z$) can be transformed to the

![Fig. 1. Schematic of the laser-induced HAZ ellipse in UD laminates.](image-url)
principal axes $(\xi_1, \xi_2, \xi_3)$ [4].

$$k_1\xi_1^2 + k_2\xi_2^2 + k_3\xi_3^2 = 1$$  \hspace{1cm} (3)

where $\xi_1$, $\xi_2$, and $\xi_3$ are coordinates of the principal axes, $k_1$, $k_2$, and $k_3$ are principal conductivities.

In this study, the workpiece (Fig. 1) is symmetric in $z$-axis, i.e. $k_{13} = k_{31} = k_{23} = k_{32} = 0$. The measured principal conductivities, as shown in Table 1, satisfy Eq. (2b). Thus $k_0$ in the desired directions can be calculated through Eqs. (2b) and (3). The derived results are listed in Table 2.

2.2. Isotherm method

In this section, an experimental method to determine the thermal conductivity is introduced. Due to the significant anisotropic thermal conductivity of the composite, the temperature rise and HAZ subject to laser irradiation will also be anisotropic, showing an elliptical shape, as illustrated in Fig. 1.

On the other hand, the heat conduction occurs as a result of the temperature gradient in workpiece and is dependent on the material thermal diffusivity ($\alpha$) and the interaction time ($t$). These define the thermal penetration depth ($\delta$) [5]

$$\delta = \sqrt{\alpha t}$$  \hspace{1cm} (4a)

The ratio between the depths in parallel and transverse directions is

$$\frac{\delta_2}{\delta_1} = \frac{r_2}{r_1} = \frac{\sqrt{\alpha_2 t}}{\sqrt{\alpha_1 t}} = \frac{\sqrt{k_2}}{\sqrt{k_1}}$$  \hspace{1cm} (4b)

For an isotherm at any given time, Eq. (4b) can be rewritten as

$$k_2 = \left(\frac{r_2}{r_1}\right)^2 k_1$$  \hspace{1cm} (4c)

and

$$k_3 = k_2$$  \hspace{1cm} (4d)

where $r_1$ and $r_2$ are the radii of the ellipse, as shown in Fig. 1. While $k_1$ is the longitudinal conductivity (in the fiber direction) determined by Eq. (1), namely the rule-of-mixture in terms of the properties of the constituents, the transverse thermal conductivities of composites ($k_2$ and $k_3$) are obtained from Eq. (4c).

In general, one can rewrite Eq. (4c) to obtain the heat
conductivity in any direction on the UD laminate

\[ k_\theta = k_1 \left( \frac{r_1}{r_\theta} \right)^2 \]  

where \( r_\theta \) is shown in Fig. 1.

If the reference value of thermal conductivity along the fiber axis is derived from Eq. (1a) or in a suitable way measured. The thermal conductivity in a generic direction can be obtained by Eq. (5).

3. Analysis of anisotropic grooving by numerical finite-difference scheme

The results of Section 2 can be used for the case of anisotropic grooving. The differential equation of heat conduction for anisotropic solids involves the cross-derivative coefficients of thermal conductivity and, therefore, a closed-form analytical solution for the extent of the HAZ cannot be derived. In this study, the numerical Finite-Difference Method (FDM) is used to predict the HAZ. When the cross-derivative terms of thermal conductivity vanish in the heat conduction equation, the analysis of heat transfer is simplified to the particular case of principal-axis grooving.

The differential equation of heat conduction for an anisotropic solid is given by Carslaw et al. (1959) [4]:

\[
\begin{aligned}
&k_{11} \frac{\partial^2 T}{\partial x^2} + k_{22} \frac{\partial^2 T}{\partial y^2} + k_{33} \frac{\partial^2 T}{\partial z^2} + (k_{12} + k_{21}) \frac{\partial^2 T}{\partial x \partial y} \\
&+ (k_{13} + k_{31}) \frac{\partial^2 T}{\partial x \partial z} + (k_{23} + k_{32}) \frac{\partial^2 T}{\partial y \partial z} + \dot{q} \\
&= \rho C \frac{\partial T}{\partial \tau}
\end{aligned}
\]

where \( k_{ij} = k_{ji} \) from the reciprocity law and \( \tau \) denotes time, \( T \) is temperature, \( C \) is specific heat of composite material, and \( \rho \) is density of composite material.

One considers a three-dimensional (3D) heat transfer in Fig. 2. The boundary conditions on the surfaces are

(1) at \( x = 0 \) and \( x = L_x \)

\[ + k_{11} \frac{\partial T}{\partial x} + k_{12} \frac{\partial T}{\partial y} + h(T - T_\infty) = 0 \]  

(7a)
where the negative and positive signs are for the two different boundaries, and $h$ is the heat convection coefficient.

The value of $h$ required in FDM is calculated for the case of vertically impinging jet from [5]

$$h = \frac{13Re^{0.5} Prn^{0.33} k_{gas}}{B}$$

where $B$ is the jet plate distance, $Re = V_{gas}d_{nozzle}/\nu$ (Reynolds number at jet exit), $Prn = \nu/\alpha$ ($\alpha$ is thermal diffusivity), $V_{gas}$ is velocity of assist gas at nozzle, $d_{nozzle}$ is nozzle diameter of assist gas, $k_{gas}$ is conductivity of assist gas, and $\nu$ is kinematic viscosity of the gas. The Bernoulli equation for a frictionless flow is used to determine the gas velocity at the nozzle exit.

The nodal equations for central and mixed partial derivatives are

$$\frac{\partial^2 T}{\partial x^2} = \frac{(T_{m+1,n,l} + T_{m-1,n,l} - 2T_{m,n,l})}{(\Delta x)^2} + O[(\Delta x)^2]$$

(9a)

where $O[(\Delta x)^2]$ is truncation error

$$\frac{\partial^2 T}{\partial x \partial y} = \frac{1}{2\Delta x} \left( \frac{T_{m+1,n+1,l} - T_{m+1,n-1,l} - T_{m-1,n+1,l} + T_{m-1,n-1,l}}{2\Delta y} \right)
+ O[(\Delta x)^2 + (\Delta y)^2]$$

(9b)

$$\frac{\partial T}{\partial t} = \frac{(T_{m,n,l+1} - T_{m,n,l})}{\Delta t} + O[\Delta t]$$

(9c)

where the subscripts $m$, $n$, and $l$ denote the $x$, $y$, and $z$ position, respectively.

The scheme equations for FDM are derived from central, forward and mixed partial formulae. The Gauss–Seidel iteration is used to solve the above difference equations.

4. Experiment

The purpose of the experiment is to compare the HAZ obtained during laser grooving at varying conditions with the predictions from the proposed methods.

4.1. Test specimen

To demonstrate the anisotropic HAZ of FRP, the specimen used is an unidirectional Carbon/Epoxy of fiber volume fraction 50%. It is laminated from unidirectionally carbon fiber-reinforced prepreg to 8 mm in thickness (64 laminas) and $20 \times 40$ mm$^2$ in area. The temperature of decomposition for the Carbon/Epoxy was determined by thermal gravimetric analysis (TGA), as shown in Fig. 3. The char temperature ($T_c = 633$ K) is defined as the matrix weight loss of 4% [6], which determines the extent of HAZ. The isotherm was observed by optical microscope.

The thermal conductivities of the samples in principal directions and at $45^\circ$ to fiber axis were measured at $23^\circ$C by the Laser Flash Method (LFM) utilizing a Holometrix Microflash instrument. This process conforms to “ASTM E146-92” for the measurement of thermal conductivity [6]. The results are given in Table 1.

4.2. Laser equipment

The laser beam (TEM$00$) was supplied by a 1000 W CO$_2$ laser and directed through an optical assembly to a focusing lens. Grooving experiments were conducted using beam power levels from 350 to 525 W with continuous beam, scanning velocities from 2.5 to 110 mm/s, and single beam pass. A coaxial N$_2$ jet was used to protect the laser lens from the debris and to provide an inert environment for laser processing.

4.3. Experimental procedure

Experimental parameters include the laser beam power...
Fig. 7. Comparison between the thermal conductivity models: (a) UD carbon/epoxy, $V_{\text{fiber}} = 53\%$, thickness = 2 mm, $P = 100$ W, $\tau = 8$ s; (b) UD carbon/epoxy, $V_{\text{fiber}} = 53\%$, thickness = 2 mm, $P = 200$ W, $\tau = 8$ s; (c) UD carbon/epoxy, $V_{\text{fiber}} = 53\%$, thickness = 4 mm, $P = 100$ W, $\tau = 8$ s; (d) UD carbon/epoxy, $V_{\text{fiber}} = 53\%$, thickness = 4 mm, $P = 200$ W, $\tau = 8$ s.
(P), traverse speed (V), power duty (Q), laser grooving direction relative to fiber axis (θ), and workpiece temperature (T₀). The experiments are divided into two parts.

### 4.3.1. Non-cryogenic grooving

Experiments were performed without liquid N₂ (only low-pressure coaxial N₂ gas was applied) to analyze HAZ varying with process parameters, i.e. beam power, power duty, and transverse velocity as well as grooving direction relative to fiber axis, as shown in Fig. 4. An energy density parameter \( PQ/V \) was used to incorporate all process conditions into one variable. This parameter correlates the total amount of energy received by a unit grooving length to machining parameters.

### 4.3.2. Cryogenic grooving

The process parameters were kept the same as above, while a cryogenic environment was established. A cooling N₂ jet connected to the pressure accumulator and a temperature acquisition unit were used. The schematic diagram of the experimental system is shown in Fig. 5. After the test specimen had reached an equilibrium temperature, i.e. the preset temperature \( T₀ \), 30, −30, and −90°C, respectively, the laser power system was turned on. At varying cryogenic temperature, HAZ was examined after laser machining.

\( W_d \) in Fig. 4 is defined as the maximum distance between the groove front surface and the char temperature isotherm parallel to top surface perpendicular to laser traverse direction. A comparison is made between laser machining with and without the cryogenic surrounding. The \( W_d \) of HAZ is measured using an optical microscope. The experimental results will be compared with the 3D numerical prediction of heat conduction in a finite domain.

### 5. Results and discussion

#### 5.1. Thermal conductivities

The experimental evaluation of the thermal conductivity based on the measurement of the HAZ isotherm of the Carbon/Epoxy composites is shown. As shown in Fig. 6, there are three values of thermal conductivity derived by different methods. Eq. (5) is the Isotherm method to
determine the thermal conductivity in generic direction when $k_1$ is known. Mukherjee et al. conducted experiments in 1990 [11] to determine the thermal conductivity of CFRP of $V_f = 55\%$. The thermal conductivity of CFRP of $V_f = 50\%$ was measured by LFM in the current study. To verify the developed Isotherm method, the CFRP of $V_f = 53\%$ is used to compare with the experimental data. The current deviation in fiber volume fraction is considered within the tolerance of measurement of thermal conductivity (5%). The comparison of thermal conductivities is shown in Fig. 6. The thermal conductivity of obtained by Isotherm method is reasonable, which lies closely to existing experimental values. Therefore, the Isotherm model is suggested as a quick and accurate method to determine the thermal conductivity in generic direction, once the longitudinal thermal conductivity is known.

Eqs. (3) and (5) represent two thermal conductivity models. The comparison between the results from Eqs. (3) and (5) is illustrated in Fig. 7. The well-known rule-of-mixture is used to predict the thermal conductivity $k_1$ parallel to fiber axis for unidirectional composite material (see Eq. (1)). The values obtained from the two model as shown in Fig. 7 will then be used in Eq. (9) to calculate the cross-sectional HAZ for comparison (as illustrated in Figs. 4, 8–12). As to thermal conductivity $k_2$ perpendicular to fiber axis, several models exist. Although Eq. (1b) is not exactly a rule-of-mixture approach, it is selected for use since it can precisely predict the experimental data of $k_2$, such as provided in Table 2.

5.2. Anisotropic grooving

In the current study, the experiment of grooving along particular axes was conducted on Carbon/Epoxy. A schematic diagram of anisotropic grooving for UD Carbon/Epoxy relative to fiber axis is shown in Fig. 4. The FDM is used to analyze the anisotropic temperature field. By the substitution of $T = T_e = 633$ K [6], and evaporation temperature ($T_v$) = 3600 K [7], and the average values of the conductivity in Fig. 7(a)–(d) determined by

![Fig. 9. HAZ in 30° grooving ($x = 5$ mm): (a) $T_0 = 30^\circ$C; (b) $T_0 = -30^\circ$C; (c) $T_0 = -90^\circ$C.](image-url)
Eqs. (3) and (5), respectively, one can solve the extent of HAZ. Since the specimen is grooved into a certain depth beneath the top surface \((z_0)\), the location of the heat source is moving at evaporation depth, which is determined by setting \(x = \text{constant}\) and by running iterative calculation of the \(y\) and \(z\) coordinates until a satisfactory convergence is reached. The size or the of HAZ was measured to evaluate the thermal conductivities in generic direction, as shown in Figs. 8–12. The extent of companies or the experimental and theoretical values or the HAZ through numerical computation and experiment is shown in Figs. 8–11. The HAZ often occurs as long as the workpiece has to be grooved at a certain level of specific laser energy. Since the parameter \(PQ/V\) plays an important role in laser machining [3,8], the data of HAZ is illustrated as a function of this parameter. The developed HAZ model (Eqs. (6)–(9)) actually involves an implicit relationship between HAZ and \(PQ/V\) (such as the terms of heat source and time). Iteration is needed to determine the extent of HAZ affected by \(PQ/V\). Figs. 8–11 shows the results in acceptable agreement with the experiment. Both thermal conductivities obtained from Eqs. (3) and (5) are used in FDM simulation for comparison. One finds that the difference in Eqs. (3) and (5) causes little concern in the simulation of HAZ, as shown in Figs. 8–11. Two models of thermal conductivity are equally acceptable in prediction of HAZ showing agreement with the experimental results.

Laser processing results in a steep temperature gradient in the vicinity of kerf. However, it decreases dramatically away from the laser beam impinging point. In the current analysis of HAZ, the major concern of temperature field lies far from the kerf rather than near the kerf. The current simulation of temperature field shows a good agreement with that of [9] except in the vicinity of kerf. Therefore, the current approach to the effect of the process temperature is reasonable. As to the effect of the interaction time, Eq. (4a) and Fig. 1 show the interaction time \((\tau)\) causes a proportional and isotropic effect on the extent of HAZ; therefore the discussions of the anisotropic thermal conductivity are independent of interaction time in the current study.

Fig. 12 reveals that the HAZ in grooving \(0^\circ\) relative to the fiber axis is narrower than that of \(90^\circ\), while \(30\) and \(60^\circ\) grooving is between the two. The reason is that carbon fiber has a much higher thermal conductivity in the
Fig. 11. HAZ in 90° grooving (x = 5 mm): (a) $T_0 = 30^\circ$C; (b) $T_0 = -30^\circ$C; (c) $T_0 = -90^\circ$C.

Fig. 12. Experimental results of HAZ in 0, 30, 60 and 90° grooving of carbon/epoxy: (a) $T_0 = 30^\circ$C; (b) $T_{>0} = -90^\circ$C.
longitudinal direction, and the heat is mainly conducted along the fiber axis than in transverse direction, hence machining in direction closer to transverse direction produces larger HAZ.

6. Conclusions

The prediction of HAZ in laser machining of anisotropic materials requires knowledge of the anisotropic thermal conductivity. In the existing literature, the heat conductivity in principal directions can be determined based on the properties of fiber and matrix. In the current study, the Eigenvalue method (analytical) and Isotherm method (experimental) were used to determine the thermal conductivity in non-principal directions. A 3D numerical analysis of HAZ using FDM and the obtained anisotropic heat conductivities for laser grooving of fiber-reinforced composite materials is derived. The laser grooving experiments conducted under various conditions. The results show that both methods for the evaluation of anisotropic heat conductivity of an UD FRP are equivalently effective. The results of both simulation and experiment reveal that grooving parallel to fiber orientation produces smaller HAZ than that of grooving perpendicular to fiber orientation, while the HAZ of non-principal-axis grooving lies between them.

References